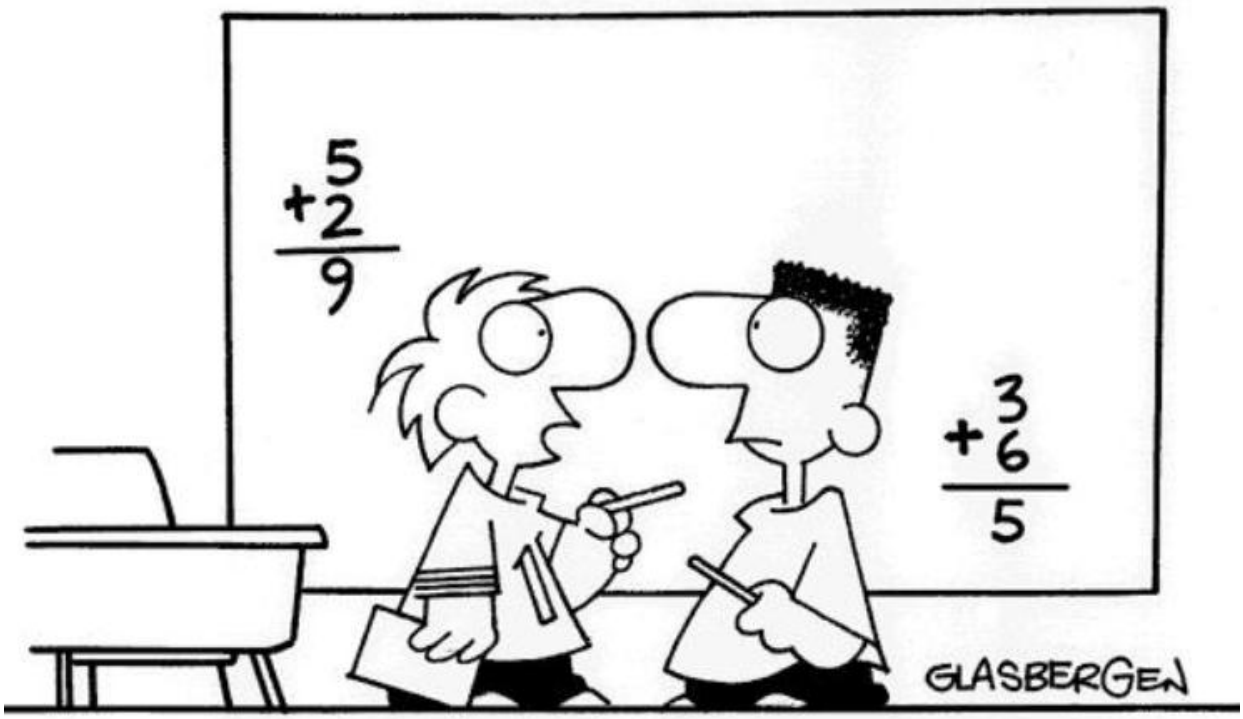
4.1 Rational Functions



"My lawyer says I can sue the school because they're violating my right to be stupid."

Graph:

Scaffolding

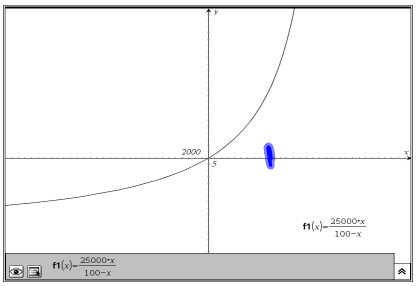
- Domain / Range
  - input (x values)
  - output (y values)
- Intercepts:
  - x int → zeros, solutions, roots
  - y int → plug in 0

think

- plot pts. (table)

In a pilot project, a rural township is given recycling bins for separating and storing recyclable products. The cost C (in dollars) for supplying bins to p% of the population is given by:

$$C = \frac{25{,}000\,p}{100-p}, \quad 0 \leq p < 100$$



* input

Rational Functions

- Rational Function:

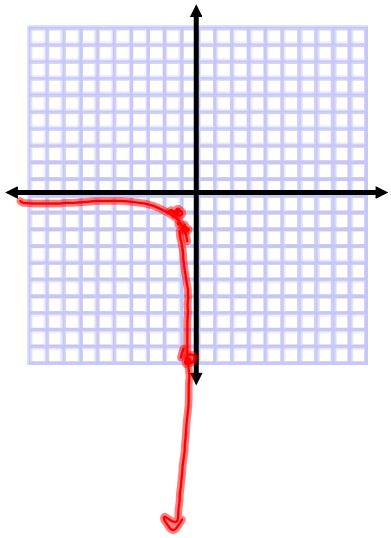
- Ex 1) Find the domain of $f(x)=\frac{2x+1}{x^2-4}$

$D:=\{x \mid x\in\mathbb{R}, x\neq 2,-2\}$

Find the domain and end behavior of:

$$f(x) = \frac{1}{x}$$

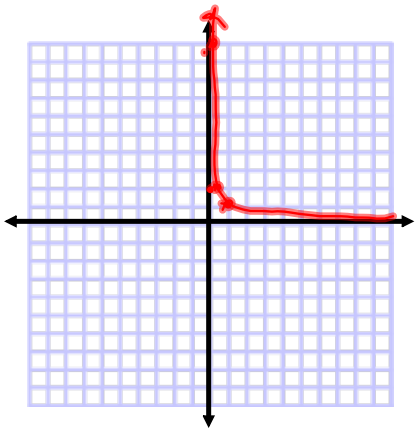
X	Y
-1	-1
-0.5	-2
-0.1	-10
-0.01	-100
-0.001	-1000
-0	-∞
→ 0	



Find the domain and end behavior of:

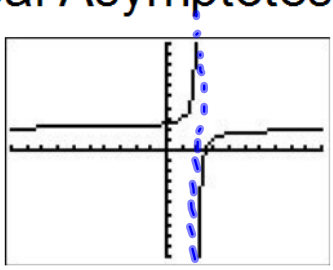
$f(x) = \frac{1}{x}$

X	Y
0←	∞
0	0
0.001	1000
0.01	100
0.1	10
0.5	2
1	1



B. Horizontal and Vertical Asymptotes

- Graph: $f(x)=\frac{2x-5}{x-2}$



Definitions:

1) The line x = a is a vertical asymptote of the graph of *f* if $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$, either from the right or from left.

2) The line y = b is a horizontal asymptote of the graph of *f* if $f(x) \to b$ as $x \to \infty$ or $x \to -\infty$.

Asymptotes of Rational Functions Rules:

1) The graph of *f* has *vertical* asymptotes at the zeros of D(x).
   denominator = 0 , simplifies & "hole" removable discontinuity
2) The graph of *f* has at most one *horizontal* asymptote determined by comparing the degrees of N(x) and D(x).
   Let the degree of numerator = n and degree of denominator =m

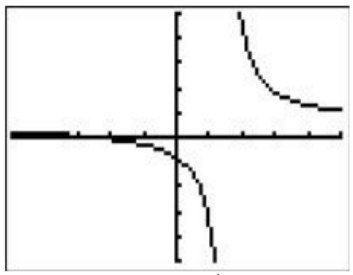
   a) If $n < m$, the graph of *f* has the line y = 0 (the axis) as a horizontal asymptote.

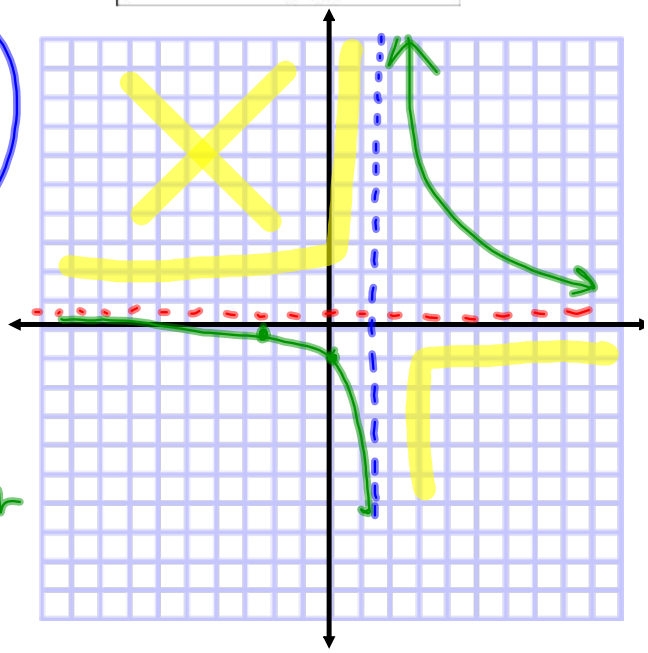
   b) If $n = m$, the graph of *f* has the line $y = a_n / b_m$ as a horizontal asymptote, where a_n is the leading coefficient of the numerator and b_m is the leading coefficient of the denominator.

   c) If $n > m$, the graph of *f* has no horizontal asymptotes.

b) $f(x)=\dfrac{2x+5}{4x-6}$ = $\dfrac{2x+5}{2(2x-3)}$

Hint: Always simplify first!!
Discuss domain.





① VA: $x=\frac{3}{2}$ (makes denom. zero)

② HA: $y=\frac{1}{2}$

③ Intercepts:

x int: $(-\frac{5}{2}, 0)$ makes numerator zero

y int $(0, -\frac{5}{6})$

c) $f(x)=\dfrac{x^2}{x+1}$ *slant

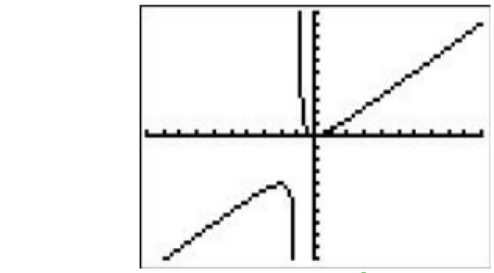
VA: $x=-1$

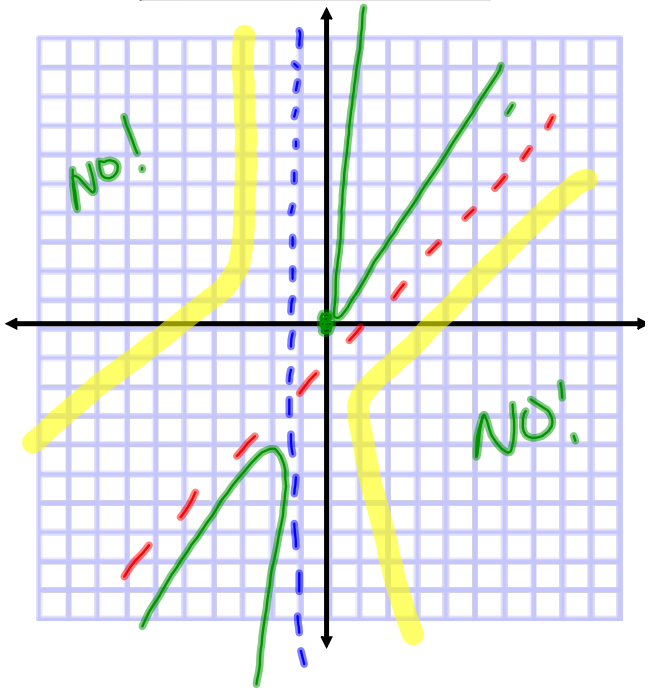
HA: ~~0~~

Intercepts:

$x \rightarrow (0,0)$

$y \rightarrow (0,0)$





Ex 2) Find any horizontal and vertical asymptotes of the following:

a) $f(x)=\frac{x+1}{x^2-1}$ $= \frac{x+1}{(x-1)(x+1)}$

VA: $x=1$

$(x=-1$ is a hole$)$

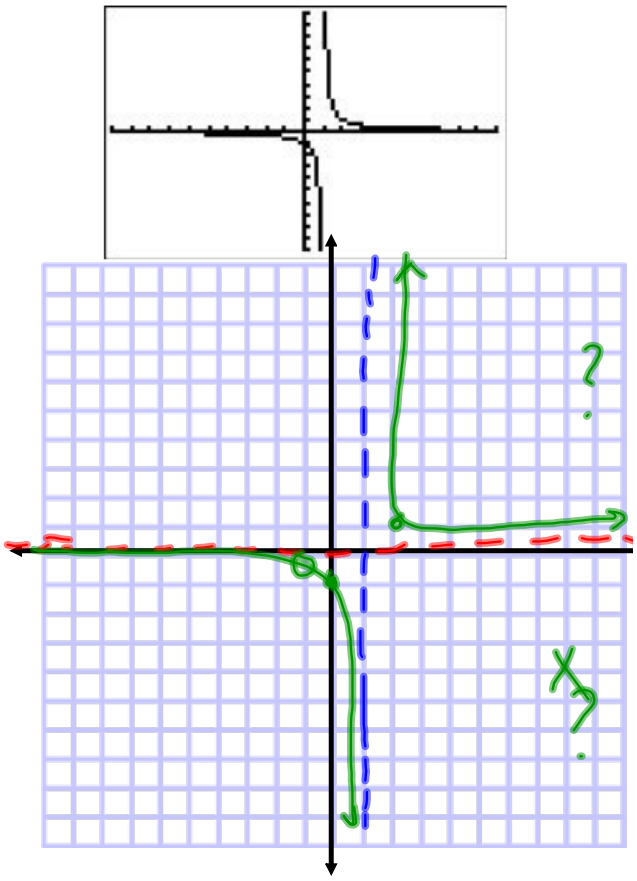
HA: $y=0$

Intercepts:

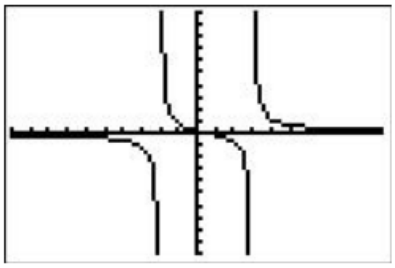
$x \to 0$

$y \to (0,-1)$

x	
2	$\frac{3}{3}$



d) $f(x)=\dfrac{2x-1}{x^2-x-6}$ $= \dfrac{2x-1}{(x-3)(x+2)}$



VA: $x=3$
$x=-2$

HA: $y=0$

intercept:

$x \to \left(\frac{1}{2}, 0\right)$ * hole on asymptote!

$y = \left(0, \frac{1}{6}\right)$

-4	$\frac{-9}{14}$
4	$\frac{7}{6}$